Reg. No. :

Question Paper Code : 30876

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024.

Fourth Semester

Computer and Communication Engineering

MA 8451 – PROBABILITY AND RANDOM PROCESSES

(Common to : Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours — Maximum : 100 marks

(Normal table is to be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A bag contains eight red balls, four green balls, and eight yellow balls. A ball is drawn at random from the bag, and it is not a red ball. What is the probability that it is a green ball?

2. Criticize the following statement: "The mean of a Poisson distribution is 7 while the standard deviation is 3".

3. List any two properties of joint distributions of X and Y.

4. Given the joint probability density function $f(x, y) = \begin{cases} e^{-x-y}, & x > 0, y > 0 \\ 0 & elsewhere \end{cases}$, what is the marginal density of X?

5. "The sum of two independent Poisson processes is a Poisson process". Prove or disprove.

6. Consider the random process $X(t) = \cos(t + \phi)$, where ϕ is uniformly distributed in the interval $-\pi/2$ to $\pi/2$. Check whether the process is stationary or not.

7. Define power spectral density function of stationary random process.

8. The power spectral density of a random process $X(t)$ is given by $S_{XX}(\omega) = \begin{cases} \pi, & |\omega| < 1 \\ 0, & elsewhere \end{cases}$. Find its autocorrelation function.

9. Mention any two properties of cross power density spectrum.

10. What is the relation between input and output of a linear time invariant system?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A student buys 1000 integrated circuits (ICs) from supplier A, 2000 ICs from supplier B, and 3000 ICs from supplier C. The student tested the ICs and found that the conditional probability of an IC being defective depends on the supplier from whom it was bought. Specifically, given that an IC came from supplier A, the probability that it is defective is 0.05; given that an IC came from supplier B, the probability that it is defective is 0.10; and given that an IC came from supplier C, the probability that it is defective is 0.10. If the ICs from the three suppliers are mixed together and one is selected at random, what is the probability that it is defective? Given that a randomly selected IC is defective, what is the probability that it came from supplier A? (8)

(ii) The discrete random variable K has the following probability mass function:

$$P_K(k) = \begin{cases} b, k = 0 \\ 2b, k = 1 \\ 3b, k = 2 \\ 0, otherwise \end{cases}.$$

(1) What is the value of b? (2)

(2) Determine the values of $P[K \leq 2]$ and $P[0 < K < 2]$ (4)

(3) Determine the cumulative distribution function of K. (2)

Or

(b) (i) Assume that the length of phone calls made at a particular telephone booth is exponentially distributed with a mean of three minutes. If you arrive at the telephone booth just as Chris was about to make a call, find the following:

(1) The probability that you will wait more than 5 minutes before Chris is done with the call.

(2) The probability that Chris' call will last between 2 minutes and 6 minutes. (8)

(ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modeled by an N (5; 16) normal random variable, X.

(1) What is the probability that a randomly selected parcel weights between 1 and 10 pounds? (4)

(2) What is the probability that a randomly selected parcel weights more than 9 pounds? (4)

12. (a) A fair coin is tossed three times. Let X be a random variable that takes the value 0 if the first toss is a tail and the value 1 if the first toss is a head. Also, let Y be a random variable that defines the total number of heads in the three tosses.

(i) Determine the joint probability mass function of X and Y. (4)

(ii) Are X and Y independent? (4)

(iii) Find the marginal probability mass functions of X and Y. (4)

(iv) Find the conditional probability of Y given X. (4)

Or

(b) Two events A and B are such that $P[A] = 1/4$, $P[B \mid A] = 1/2$ and $P[A \mid B] = 1/4$. Let the random variable X be defined such that $X = 1$ if event A occurs and $X = 0$ if event A does not occur. Similarly, let the random variable Y be defined such that $Y = 1$ if event B occurs and $Y = 0$ if event B does not occur.

(i) Find $E[X]$ and the variance of X. (4)

(ii) Find $E[Y]$ and the variance of Y. (4)

(iii) Find ρ_{XY} and determine whether or not X and Y are uncorrelated. (8)

13. (a) There are 2 white marbles in urn A and 3 red marbles in urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are inter changed. Let the state a_i of the system be the number of red marbles in A after i changes. What is the probability that there are 2 red marbles in A after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn A? (16)

Or

(b) (i) Suppose that a customer arrives at a bank according to Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 min.

(1) exactly 4 customers arrive and (4)

(2) more than 4 customers arrive. (4)

(ii) Prove that the random process $X(t) = A\cos(\omega t + \theta)$ where ω and θ are constants, A is a random variable with zero mean and variance one and θ is uniformly distributed on the interval $(0, \pi)$. Assume that the random variable A and theta are independent. Is $X(t)$ is mean-ergodic process? (8)

14. (a) (i) Two random processes $X(t)$ and $Y(t)$ are defined as follows: $X(t) = A\cos(\omega t + \theta)$, $Y(t) = B\sin(\omega t + \theta)$ where A, B and ω are constants and θ is a random variable that is uniformly distributed between θ and 2π. Find the cross-correlation function of $X(t)$ and $Y(t)$. (8)

(ii) A random process is defined by $X(t) = K\cos wt, t \geq 0$ where w is a constant and K is uniformly distributed between 0 and 2. Determine the following:

(1) $E[X(t)]$ (2)

(2) The autocorrelation function of $X(t)$ (3)

(3) The auto-covariance function of $X(t)$. (3)

Or

(b) (i) Two random processes $X(t)$ and $Y(t)$ are both zero-mean and wide-sense stationary processes. If we define the random process $Z(t) = X(t) + Y(t)$, determine the power spectral density of $Z(t)$ under the following conditions:

(1) $X(t)$ and $Y(t)$ are jointly wide-sense stationary.

(2) $X(t)$ and $Y(t)$ are orthogonal. (8)

(ii) Two jointly stationary random processes $X(t)$ and $Y(t)$ have the cross correlation function given by : $R_{XY}(T) = 2e^{-2T}$, $T \geq 0$ Determine cross-power spectral density $S_{XY}(w)$. (8)

15. (a) A random process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t \geq 0$. If the autocorrelation function of the process is $R_{XY}(\tau) = 2e^{-2|\tau|}$, Determine cross correlation function $R_{XY}(T)$ between the input process $X(t)$ and the output process $Y(t)$ and the cross correlation function $R_{YX}(T)$ between the output process $Y(t)$ and the output process $X(t)$.

Or

(b) $X(t)$ is a wide-sense stationary process that is the input to a linear system with the transfer function $H(\omega) = \dfrac{1}{a + j\omega}, a > 0$. If $X(t)$ is a zero-mean white noise with power spectral density $\dfrac{N_0}{2}$, determine the following:

(i) The impulse response $h(t)$ of the system. (4)

(ii) The cross-power spectral density $S_{YX}(\omega)$ of $Y(t)$ and $X(t)$. (6)

(iii) The power spectral density $S_{YX}(\omega)$ of the output process. (6)
